RENEWABLE ENERGY
CDT in Photovoltaics
Examples Sheet

Waves and tide

1 Waves. By estimating the potential energy and kinetic energy in a wavelength, derive the power per unit wave-crest in a wave of amplitude h: $P = \frac{1}{2}\rho g h^2 v$, where v is the wave speed. Estimate the power of deep-water waves per unit length of Atlantic coastline.

[Hint: use the dispersion relation (frequency-wavelength relation) $\omega^2 = gk$ to find the speed v of a wave of the required period, using your seaside experience to estimate the period.]

[Detail for perfectionists: the energy in a wave travels at the group velocity, not the phase velocity.]

2 Tide-pools. Assume that a tidal facility generates power at low tide (when letting water out of the pool) and at high tide (when letting it back in). Estimate the power per unit area of tide pool for a tide pool in a British estuary.

3 Tide-farms. The tidal currents off the Isle of Wight are at most 3.1 m/s. Assuming that underwater tide-farms could be made using the same principles as onshore wind-farms, estimate the power per unit sea-floor area of a tide farm in that location.

4 Upper limit for tidal power. Assume that tides approaching the British Isles from the Atlantic can be modelled as travelling waves, like the waves of question 1, except with much longer wavelength. This wavelength is much bigger than the depth, so these are shallow water waves, for which the dispersion relation is $\omega/k = (gd)^{1/2}$ where d is the water depth. Estimate the total power per unit length of coastline of tidal waves approaching the British Isles, assuming that out at a depth $d = 100\,\text{m}$, the Atlantic tidal amplitude is 1 or 2 metres.

Exergy

5 Assume an environmental temperature of 10 °C, and standard pressure. What's the exergy of

(a) $1\,\text{m}^3$ (1000 kg) of water at 100 °C?

(b) $1\,\text{m}^3$ (1000 kg) of water at 0 °C?

(c) 1000 kg of steam at 100 °C?

(d) 1000 kg of ice at 0 °C?

(e) 1000 kg of air, at room temperature, that has been separated into 200 kg of oxygen and 800 kg of nitrogen, both at atmospheric pressure?

Give all the answers in joules or MJ, and also in kWh (3.6 MJ = 1 kWh).

How does the exergy of 1000 kg of water at 100 °C (part (a), above) compare with the heat it contains? So, in exergy terms, what is the efficiency of a "perfect" boiler that sets fire to natural gas and converts 100% of the chemical energy into heat energy in 100 °C water?

[The heat capacity of water is near enough 4.2 kJ/kg/K at all temperatures.]

6 Define the concept of *exergy* and explain its usefulness in practical energy problems.

A thermal storage system uses a spherical tank of fluid, of radius r. The fluid is initially at the ambient temperature of 20 °C, and energy is stored by heating the fluid electrically. When required, electricity is regenerated by an ideal generator attached to an ideal heat engine using the fluid as the hot source, and a local river at 10 °C as the cold sink. The fluid is continuously stirred so it maintains a uniform temperature. Heat losses occur from the surface of the sphere, at a rate per unit area of $\lambda\,\Delta T$, where ΔT is the temperature difference between the fluid and the ambient. The heat capacity per unit volume of the fluid, c, is $5 \times 10^6\,\mathrm{J\,m^{-3}\,K^{-1}}$, and $\lambda = 6\,\mathrm{W\,m^{-2}\,K^{-1}}$.

Show that, during storage, the temperature of the fluid decays exponentially towards the ambient temperature with a time constant of $\frac{1}{3}cr/\lambda$.

A first system with $r = 4$ m is heated to 120 °C. Find the temperature after one week of storage, and calculate (using exergy or otherwise) the amount of electrical energy that can subsequently be recovered using the heat engine to return the fluid to the ambient temperature.

In an attempt to reduce the effect of heat losses, the same amount of initial energy is stored in a second, similar, system with twice the volume ($r = 5.0$ m), at a temperature of 70 °C. Repeat the calculation above to detemine the electrical energy that can be regenerated after one week.

Explain qualitatively why less energy is regenerated in this case.

Briefly explain the Stirling cycle, and discuss the potential benefits of a Stirling engine based on this cycle as a heat engine for the system described above.

Storage

7 Make estimates of the practical energy density (in kWh per m^3, and, if appropriate, in kWh per kg) of the following possible forms of stored energy:

- Electrical energy (batteries). [Hint: one way to think about the basic mechanism of an electrical cell is that it moves electrons from a close relationship with one atom to a closer relationship with another.]

- Magnetic energy (superconducting solenoids) [What limits a superconducting solenoid? Is there a maximum field?]

- Inertial energy (flywheels) [Hint: the limit involves the strength Y (yield stress) of the material from which the wheel is made – that's a maximum force per unit area that it can withstand. Steel has density $\rho = 7800\,\mathrm{kg/m^3}$ and strength $Y = 1800\,\mathrm{MN/m^2}$; carbon fibre reinforced polymer has density $\rho = 1500\,\mathrm{kg/m^3}$ and strength $Y = 2400\,\mathrm{MN/m^2}$.]

- Gravitational potential energy (hydroelectric pumped storage)
- Compressed air
- Electricity storage using heat or cold. [A local company, *Isentropic*, stores electricity by using a reversible heat pump to warm up or cool down gravel. Assuming a maximum temperature of 500 °C, how big a container full of gravel would be required to store 10 GWh of electricity?]

8 Many forms of renewable energy are intermittent and therefore storage is often required. Estimate how much storage might realistically be required to complement significant exploitation of on-shore wind farms in the UK. Discuss the feasibility of providing this through hydroelectric storage.

Energy generation, transport and use

9 Transport efficiency: Freight transport efficiencies are often measured in kWh per ton-km. (For example, if a truck carrying 3 tons of stuff drives 4 km and uses 10 kWh of fuel, we say it delivered transport at a cost of $10/(3 \times 4)$ kWh per ton-km.) What are the dimensions of "1 kWh per ton-km"? Express the gravitational constant ($g = 10\,\mathrm{m/s/s}$) in kWh per ton-km. When you see the water gushing behind a big boat's thrusters, water transport doesn't look especially efficient. Yet freight-transport by water requires only 0.1 kWh per ton-km, or perhaps even less*, whereas trucks cost roughly 1 kWh per ton-km. Why is water transport so efficient? [(*) Container ships travel at 25 knots, and a 5000-container ship uses 44 MW of delivered power from its engine. Each container can transport 13 tonnes. Marine diesel engines are 50% efficient.]

10 Assuming that a truck delivers freight at a cost of 1 kWh per tonne-km, what fraction of the energy in coal is used if the coal is transported 1000 km by road? [Hint: coal has an energy density of 26.7 GJ/ton (7400 kWh/t).] Answer: 14% (for a one-way journey).

How deep would a coal-mine have to be before you would worry about the gravitational energy cost of lifting the coal to the surface?

11 What is the fractional energy lost when crude oil is shipped 1000 km by tanker? [Hint: An oil tanker carrying 40 000 t of cargo travelling at 15.5 knots needs 8.2 MW of engine power. Marine diesel engines are roughly 50% efficient. Crude oil occupies a volume of 1192 litres per tonne.] Answer: 0.1%.

12 Heating: Take a building that needs 40 kWh per day to stay warm in winter. A proposed eco-solution is to collect summer heat (perhaps using solar water-heating panels) and store it somewhere, then provide most of the winter heat from this store. How big a volume of rock or water would be required to store two months' worth of winter heat loss? How does this compare with the volume of the bricks in a typical house? [Hints: Conductivity of granite: 2.1 W/m/K. Specific density of granite: 2.5. Heat capacity of granite: 820 J/kg/K. Thermal conductivity of water: 0.6 W/m/K. density 1000 kg/m^3; heat capacity 4200 J/kg/K.]

Imagine that the heat is stored in a block of rock 4 m by 4 m by 4 m in the ground. For insulation, the rock is simply surrounded by more rock. If 1000 kWh of heat is stored in the block in July, roughly how much of that heat will still be there in January? Hint: a single spike of heat diffuses; the impulse response is a Gaussian that grows with time. The width of the Gaussian can be deduced by dimensional analysis. The width depends on time, heat capacity, density, and conductivity. What is it? [Further hint once you've done the dimensional analysis: the missing constant is sqrt(2).] [Answer: width = $\sqrt{2\kappa t/C\rho}$ where κ is conductivity.]

Hydrogen

13 Discuss the origin of the dependence of cell potential on concentration in the Nernst equation. If the emf of a hydrogen fuel cell is 1.21 V at 300 K, calculate its value at 700 °C. [The standard cell potential is 1.23 V. Neglect any temperature dependence of activities.] If hydrogen is to be produced by electrolysis of water, is there any advantage in carrying this out at an elevated temperature?

14 The hydrogen in the Honda Clarity Fuel Cell Vehicle is stated to be stored in tanks of volume 171 l at a pressure of 5000 psi (3.4×10^7 Pa). Estimate the energy required to compress the hydrogen from atmospheric pressure. What fraction of the calorific value of the hydrogen does this represent?

15 The nuclear spin state of H_2 affects the allowed rotational modes of the molecule, and has important effects at low temperatures. Explain why spin-parallel, ortho-H_2 is 75% abundant and para-H_2 25% abundant at room temperature, but at low temperatures, 100% para-H_2 is expected. The interconversion from ortho to para states is exothermic (estimate the energy release per molecule at low temperatures) and can be slow, occurring days after liquefaction. What fraction of liquid hydrogen initially prepared as 75:25 ortho:para will be boiled off during conversion to all-para? [The latent heat of vaporisation of liquid hydrogen is 450 kJ/kg.]